A Simplified Method of Estimating the Response of Light Aircraft to Continuous Atmospheric Turbulence

E. L. PEELE* AND ROY STEINER†
NASA Langley Research Center, Hampton, Va.

Power spectral density techniques employed in the design of large flexible civil aircraft for atmospheric turbulence are rather complex. For the small rigid aircraft, this degree of complexity may be unwarranted. Consequently, a procedure for determining the gust response based on two rigid-body degrees of freedom was investigated. Using certain simplifying assumptions, formulas are derived and presented for calculating the vertical and lateral gust response to independently applied vertical and lateral gusts. The formulas utilize parametric charts of "response integrals" that are functions of the aircraft reduced frequency, the damping ratio, and a gust scale parameter. The response quantities for a given set of aircraft parameters can be obtained by substituting into appropriate formulas values of two response integrals read from the charts. Sample numerical calculations are included which illustrate the use of the simplified procedure.

Nomenclature

$\bar{A}_y$	= airplane response parameter relating rms input and yth output values
a	= unsteady lift force attenuation factor
b	= wing span
C_1, C_2, etc.	= tail load response parameters defined in text
$\bar{c}$	= streamwise chord of lifting surface
$C_{L\alpha}, C_{m\alpha}$, etc.	= aerodynamic stability derivatives
e	= exponential function
g	= acceleration due to gravity
$H(\omega)$	= frequency response function
I_{ii}	= mass moment of inertia about ith axis
i	= $(-1)^{1/2}$
K_g, K_Φ	= gust alleviation factors
k	= reduced frequency, $\bar{c}\omega/2U$
L	= scale of turbulence
$M_w, M_{\dot{w}}, M_q$	= moment coefficients about y axis
m	= mass of airplane
N_{0_y}	= average number of positive slope zero crossings of y, the response quantity
N_v, N_r, N_w	= moment coefficients about z axis
q	= dynamic pressure, $q = \frac{1}{2}\rho U^2$
q	= pitch velocity
$R_{\beta(j)}$	= jth "response integral," $j = 0,2,4,6$
r	= yaw velocity
S	= area of aerodynamic surface
sk_0	= nondimensional gust scale, $sk_0 = L\omega_0/U$
U	= average velocity along x axis
v	= perturbation velocity along y stability axis
w	= perturbation velocity along z stability axis
W	= airplane weight
x,y,z	= Cartesian coordinates
Y,Z	= force coefficients along y,z axes
α	= plunge angle $\approx w/U$
β	= sideslip angle $\approx v/U$ when subscript
β	= frequency ratio $= \omega/\omega_0$
γ	= damping factor
Δn	= incremental vertical load factor
ζ	= damping ratio
θ	= pitch angle
ρ	= air density
σ_j	= rms value of jth quantity
Φ	= power spectrum
ϕ	= unsteady lift function, vertical gusts
Ψ	= unsteady lift function, lateral gusts
ψ	= yaw angle
Ω	= spatial frequency, ω/U
ω	= circular frequency
ω_0	= undamped natural frequency

Presented as Paper 69-766 at the AIAA Aircraft Design and Operations Meeting, Los Angeles, Calif., July 14–16, 1969; submitted July 28, 1969; revision received January 26, 1970.

* Aerospace Technologist, Atmospheric Physics Branch, Dynamic Loads Division.

† Assistant Head, Atmospheric Physics Branch, Dynamic Loads Division.

Introduction

THE analytical study of the response of flight vehicles to atmospheric turbulence has, in the past 10 years, been progressing from the use of the discrete gust velocity method[1] toward the use of spectral density techniques. Following the many research investigations on the applications of spectral techniques to turbulence problems, an extensive investigation was made for the development of a power spectral gust design procedure for civil aircraft.[2,3] Although such a procedure may be simple in concept, many of the details may be rather involved and more complicated than warranted for some structures. It is felt, therefore, that a simplified procedure, following the methods given,[4] could be established for estimating the response of small rigid aircraft to continuous turbulence.

Certain simplifying assumptions were made which permitted the airplane response to gust forces to be based on two rigid-body degrees of freedom. An analysis of this system indicates that the airplane response could be defined as functions of a gust wave number, the airplane damping ratio, and the undamped natural reduced frequency. Moreover, the study revealed that the response equations for both vertical and lateral motion contain the same sets of integrals. The evaluation of these integrals referred to as "response integrals" constitutes the major computational effort involved in the spectral density analysis. Consequently, the response integrals were evaluated over a limited range of the nondimensional parameters believed to correspond to those for most small rigid aircraft, say of the executive type. The purpose of this paper is to present a concise method of estimating the response of a rigid airplane to continuous turbulence based on the use of charts of "response integrals." In the following discussion, the pertinent response quantities are reviewed and the normal load factor and vertical tail side load equations formulated. The use of the parametric charts to perform response calculations is illustrated for a representative executive aircraft.

Derivation of PSD Gust Response Formulas

The procedures for determining the airplane response to atmospheric turbulence, as presented,[5] involve the use of two quantities, $\bar{A}$ and N_0. In both Ref. 5 and this paper, atmospheric turbulence is considered to be a stationary Gaussian random process. $\bar{A}$ is defined as the ratio of the rms value of a particular output response quantity to rms input gust velocity obtained by summing the output power spectral density function over all frequencies.

$$\bar{A} = \frac{\sigma_y}{\sigma_g} = \left[\int_0^\infty \Phi_y(\omega)d\omega/\sigma_g^2\right]^{1/2} \tag{1}$$

In the previous expression, Φ_y is the power spectral density (PSD) of the particular response quantity y, which is found from an input-output equation:

$$\Phi_y = \Phi_g(\omega)|H_y{}^g(\omega)|^2 \tag{2}$$

Equation (2) states that the power spectrum of the response of a linear system is given by the product of the input power spectrum $\Phi_g(\omega)$ and the system frequency response function $H_y{}^g(\omega)$. The second response quantity N_0 is referred to as a characteristic frequency. For narrow band processes, it is related to the resonant frequency and is the average number of times per second that the response crosses zero with a positive slope. N_0 is given by the following:

$$N_0 = \frac{1}{2\pi\sigma_y}\left[\int_0^\infty \omega^2\Phi_y(\omega)d\omega\right]^{1/2} \tag{3}$$

The computation of $\bar{A}$ and N_0 can be very complex. The degree of this complexity depends both on the form of the input spectrum as well as the airplane frequency response functions. For example, the analysis of a large flexible aircraft might require a two-dimensional gust representation, the inclusion of numerous elastic modes, consideration of combined stresses, and so forth. However, the response of certain classes of aircraft may be estimated to an acceptable degree without including the above requirements. The classes of aircraft considered here are limited to those for which the following assumptions are valid: 1) The turbulence scale is large compared to the airplane size. 2) The turbulence is homogeneous and isotropic; that is, the same power spectra are used for both vertical and lateral gusts. 3) The unsteady lift function can be approximated with a single attenuation function applicable to wide ranges of sweep angle, aspect ratio, and Mach number and are applicable to both lift and moment. 4) The control surfaces are fixed. 5) The aircraft does not roll when responding to lateral gusts and the Dutch roll mode can be treated as if it were a flat yawing maneuver. 6) The effects of elastic deformation are negligible.

Input Power Spectrum

For this analysis, the power spectrum of the gust input $\Phi_g(\omega)$ employed is that shown in Ref. 5 to best represent the vertical and lateral atmosphere turbulence and is generally referred to as the Von Kármán spectrum.

$$\Phi_g(\omega) = \Phi_{w_g}(\omega) = \frac{\sigma^2_{w_g}L}{\pi U}\,\frac{1 + \frac{8}{3}(1.339L\omega/U)^2}{[1 + (1.339\,L\omega/U)^2]^{11/6}} \tag{4}$$

Frequency Response Functions

The frequency response functions required by the input-output relation, Eq. (2), are obtained by solving the equations of motion of the airplane for independently applied gust forces resulting from unit sinusoidal vertical and lateral gust velocities. The approach is illustrated in the derivation of response functions for the normal load factor and vertical tail side load. The equations of motion,[6] are based on the assumptions commonly made in stability analyses of rigid airplanes. In stability analyses, the airplane's motion is described by six coordinates, three translations, and three rotations referenced to the so-called stability axes. If a plane of symmetry is assumed to exist, the terms coupling longitudinal and lateral motion can be neglected and for linearized equations, the six-degree-of-freedom system is separated into two groups, the longitudinal and the lateral. Consequently, each system can now be analyzed separately.

Normal-load factor

The three degrees of freedom comprising the aircraft longitudinal motion are vertical translation (plunge), pitch rotation, and horizontal translation in the direction of flight. Perturbations in horizontal translation have been shown to have little effect on pitching or plunging. The longitudinal case is representable, therefore, as a two-degree-of-freedom system. The equations of motion as given in Ref. 6 are

$$\begin{bmatrix} i\omega - Z_w & -i\omega U \\ -i\omega M_{\dot{w}} - M_w & -\omega^2 - i\omega M_q \end{bmatrix}\begin{Bmatrix} w \\ \theta \end{Bmatrix} = \phi(\omega)\begin{Bmatrix} Z_w' \\ M_w' \end{Bmatrix} w_g \tag{5}$$

where

$$Z_w = -(qS/mU)C_{L\alpha},\ M_w = (qS\bar{c}/I_{yy}U)C_{m\alpha}$$

$$M_{\dot{w}} = (qS\bar{c}^2/2I_{yy}U^2)C_{m\dot{\alpha}},\quad M_q = (qS\bar{c}^2/2I_{yy}U)C_{m_q}$$

In Eq. (5), $\phi(\omega)$ represents the manner in which the aerodynamic forces and moments vary with the frequency of the gust input velocity. A discussion of a suitable choice of $\phi(\omega)$, the unsteady lift function, will be given subsequently.

The frequency response functions are obtained by solving Eq. (5) for w,θ for a unit sinusoidal gust velocity w_g as follows:

$$\begin{Bmatrix} w \\ \theta \end{Bmatrix} = \frac{\begin{bmatrix} -\omega^2 - i\omega M_q & i\omega U \\ i\omega M_{\dot{w}} + M_w & i\omega - Z_w \end{bmatrix}\begin{Bmatrix} Z_w' \\ M_w' \end{Bmatrix}\phi(\omega)}{i\omega[-\omega^2 - i\omega(M_q + Z_{\dot{w}} + UM_{\dot{w}}) + (Z_wM_q - UM_w)]} \tag{6}$$

The frequency response functions are then

$$H_w{}^{w_g}(\omega) = \frac{[(-\omega^2 - i\omega M_q)Z_w' + i\omega UM_w']\phi(\omega)}{i\omega[-\omega^2 - i\omega(M_q + Z_w + UM_{\dot{w}}) + (Z_wM_q - UM_w)]} \tag{7}$$

$$H_\theta{}^{w_g}(\omega) = \frac{[(i\omega M_{\dot{w}} + M_w)Z_w' + (i\omega - Z_w)M_w']\,\phi(\omega)}{i\omega[-\omega^2 - i\omega(M_q + Z_w + UM_{\dot{w}}) + (Z_wM_q - UM_w)]} \tag{8}$$

These equations can be simplified and put into another form if it is assumed that the quasi-steady aerodynamic forces resulting from airplane motion equal those of the gust input, that is, if $Z_w = Z_w'$ and $M_w = M_w'$, and the following notation is employed:

$$\beta = \omega/\omega_{0\alpha},\ 2\zeta_\omega\omega_{0\alpha} = -(M_q + Z_w + UM_{\dot{w}})$$
$$\omega_{0\alpha} = (Z_wM_q - UM_w)^{1/2},\ \gamma_\alpha = (M_q + Z_w + UM_{\dot{w}})/Z_w \tag{9}$$

Equations (7) and (8) are now

$$H_w{}^{w_g}(\beta) = -2[(\zeta\beta/\gamma)i + 1]\phi(\omega_0\beta)/[1 - \beta^2 + i2\zeta\beta] \tag{10}$$

$$H_\theta{}^{w_g}(\beta) = (1/U)\{4(\zeta^2/\gamma)[1 - (1/\gamma)] - 1\}\,\phi(\omega_0\beta)/(1 - \beta^2 + i2\zeta\beta) \tag{11}$$

A response parameter of major interest is the incremental normal load factor $\ddot{z}/g$. Since the stability axes are rotating, the absolute acceleration is given by $-\dot{w} + U\dot{\theta}$. The frequency response of $\ddot{z}$ will be the sum of the frequency response

$\dot{w}$ and $U\dot{\theta}$ or

$$H_{\ddot{z}/_g}{}^{w}(\beta) = \frac{(2\zeta\omega_0/g\gamma)\{-\beta^2 + i2\zeta[1-(1/\gamma)]\beta\}}{1-\beta^2+i2\zeta\beta}\,\phi(\omega_0\beta) \quad (12)$$

Two-degree-of-freedom lateral motion

In order to simplify the estimate of the lateral response, it is desirable to reduce the normally considered three-degree system to a two-degree system. Previous work[7–10] indicated that this simplification is possible. It was assumed, therefore (based on the references) that 1) airplane does not roll; 2) lateral response is due to side gust only; and 3) side force due to yawing velocity is negligible.

Since the airplane types of parameters considered in the references may not have included the airplane type considered here, an additional verification was made of the applicability of these assumptions. This investigation included calculations for the three-degree-of-freedom model, the two degrees of freedom, and the two degrees of freedom with yaw side force omitted. An indication of the generality of the assumptions for the range of parameters considered here were obtained by varying the ratios of inertial and aerodynamic yaw-roll coupling terms. On the basis of this study, the lateral response equations are derived for the two degrees of freedom, yaw, and sideslip. In matrix form, the equations of motion are

$$\begin{bmatrix} i\omega - Y_v & i\omega U \\ -N_v & -\omega^2 - i\omega N_r \end{bmatrix} \begin{Bmatrix} v(\omega) \\ \psi(\omega) \end{Bmatrix} = \Psi(w) \begin{Bmatrix} -Y_v' \\ -N_v' \end{Bmatrix} v_g \quad (13)$$

where

$$Y_v = \frac{\rho US}{2m} C_{y\beta},\; N_v = \frac{\rho USb}{2I_{zz}} C_{n\beta},\; N_r = \frac{\rho USb^2}{4I_{zz}} C_{nr}$$

In Eq. (13), the function $\Psi(\omega)$ represents an approximation for the unsteady lift force and moment variation with the frequency of the lateral gust velocity $v_g(\omega)$. It is assumed, hence forth, that $|\Psi(\omega)|^2 = |\phi(\omega)|^2$.

Solutions for the variables $v(\omega)$ and $\psi(\omega)$ are obtained from Eq. (13) by assuming a unit gust velocity $v_g(\omega)$. Notice that the longitudinal and the lateral response are described by equations of the same form.

Total side load on vertical tail

The total side load acting on the vertical tail is defined as the lateral shear that would be measured by strain gages mounted at the base of the vertical tail. This load is the summation of the aerodynamic forces resulting from both the motion of the airplane and the gust input and the inertia load resulting from the acceleration response of the aircraft.

Since the purpose of this paper is to develop techniques that will enable relatively simple determinations of airplane response, the inclusion of contributory factors having negligible effect on the response was deemed undesirable. A study was therefore made to determine the relative contribution of the inertial loading to total load using several representative configurations.

On the basis of the study, it was concluded that the inertial effects are negligible. Therefore, a vertical tail side load equation is derived with the inertia terms omitted. The side load equation is

$$Y_{VT}(\omega) = (qS/U)(C_{y\beta})_{VT}v(\omega) + (qS/U)(C_{y_r})_{VT}b\dot{\psi}(\omega) - (qS/U)(C_{y\beta})_{VT}v_g\Psi(\omega) \quad (14)$$

The frequency response function for the tail load is obtained by substituting into Eq. (14) values for $v(\omega)$ and $\psi(\omega)$ from Eq. (13). The function when expressed in terms of the nondimensional frequency ratio β, the critical damping ratio ζ, and the damping factor γ is

$$H_{yVT}{}^{v_g}(\beta) = \frac{qS(C_{y\beta})_{VT}}{U}\left[\frac{\beta^2 + iC_2\beta}{1-\beta^2+i2\zeta_\beta\beta}\right]\Psi(\omega) \quad (15)$$

where

$$C_2 = -2\zeta_\beta\left(1-\frac{1}{\gamma_\beta}\right) + j_{0\beta}\frac{b}{\bar{c}}\left(\frac{C_{yr}}{C_{y\beta}}\right)_{VT} \times \left[4\frac{\zeta_\beta{}^2}{\gamma_\beta}\left(1-\frac{1}{\gamma_\beta}\right) - 1\right]$$

$$\beta = \omega/\omega_{0\beta},\quad 2\zeta_\beta\omega_{0\beta} = -(N_r + Y_v)$$

$$\omega_{0\beta} = (N_rY_v + UN_v)^{1/2},\quad \gamma_\beta = (N_r + Y_v)/Y_v$$

Response Factor

The frequency response functions, Eqs. (12) and (15), together with the input gust spectrum Eq. (4) are substituted into Eq. (1) to obtain response factors $\bar{A}$. For example, for the normal load factor,

$$\bar{A}_n{}^2 = \frac{4\zeta^2\omega_0{}^3L}{g^2\pi\gamma^2U}\int_0^\infty \times \frac{e^{-ak_0\beta}\beta^2[\beta^2 + 4\zeta^2(1-1/\gamma)^2][1+\frac{8}{3}(1.339\,sk_0\beta)^2]}{[(1-\beta^2)^2 + 4\zeta^2\beta^2][1+(1.339\,sk_0\beta)^2]^{11/6}}\,d\beta \quad (16)$$

In the previous equation, the input power spectra [Eq. (4)] has been written in terms of the frequency ratio β, the reduced frequency $k_0 = \bar{c}\omega_0/2U$, and the relative gust scale parameter $s = 2L/\bar{c}$. In addition, the magnitude of the unsteady lift function, $|\phi(\omega)|^2$ has been approximated by a simple exponential function written in terms of the reduced frequency $k = \bar{c}\omega/2U$,

$$|\phi(k)|^2 = e^{-ak} = e^{-ak_0\beta} \quad (17)$$

This form was suggested following a numerical analysis of various planforms and Mach numbers, using the kernel function procedure.[11] It might be noted that the inclusion of the effects of compressibility and finite span on the unsteady lift function was recommended in Ref. 10. In the previous form, these effects are characterized by the parameter a which is determined numerically for a given configuration by fitting Eq. (17) to the calculated values of $|\phi(k)|^2$.

Values of the response factor $\bar{A}_n$ can now be calculated for given values of the aircraft characteristics a, k_0, ζ, γ, and the relative gust scale parameter s by evaluating the infinite integral. It was found convenient to separate Eq. (16) into two integrals,

$$\bar{A}_n{}^2 = \frac{4\zeta^2\omega_0{}^2}{g^2\gamma^2}\left\{\frac{sk_0}{\pi}\int_0^\infty \times \frac{\beta^4e^{-ak_0\beta}[1+\frac{8}{3}(1.339\,sk_0\beta)^2]}{[(1-\beta^2)^2+4\zeta^2\beta^2][1+(1.339\,sk_0\beta)^2]^{11/6}}\,d\beta + 4\zeta^2\left(1+\frac{1}{\gamma}\right)^2\frac{sk_0}{\pi}\int_0^\infty \times \frac{\beta^2e^{-ak_0\beta}[1+\frac{8}{3}(1.339\,sk_0\beta)^2]}{[(1-\beta^2)^2+4\zeta^2\beta^2][1+(1.339\,sk_0\beta)^2]^{11/6}}\,d\beta\right\} \quad (18)$$

Notice that the integrals are now reduced to functions of three variables, ak_0, sk_0, and ζ. These integrals, referred to as "response integrals," are in general form:

$$R_{\beta(j)} = \frac{sk_0}{\pi}\int_0^\infty \times \frac{\beta^{(j)}e^{-ak_0\beta}[1+\frac{8}{3}(1.339\,sk_0\beta)^2]}{[(1-\beta^2)^2+4\zeta^2\beta^2][1+(1.339\,sk_0\beta)^2]^{11/6}}\,d\beta,\quad (j) = 0,2,4,6 \quad (19)$$

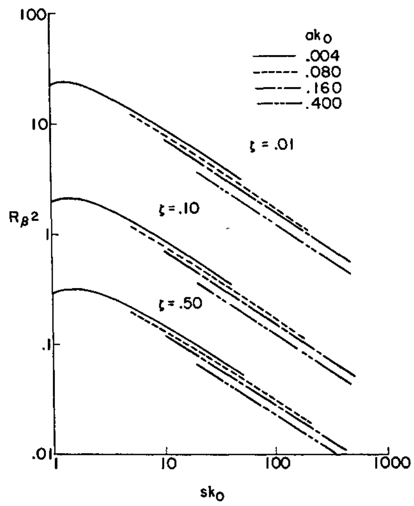

Fig. 1 Response integral, R_{β^2}.

Parametric charts have been prepared for these integrals and will be discussed in the next section. But first let us return to the response equations. Equation (1) can be written for the normal load factor in terms of response integrals as follows:

$$\bar{A}_n = 2\zeta\omega_0/g\gamma\{R_{\beta^4} + 4\zeta^2[1 - (1/\gamma)]^2 R_{\beta^2}\}^{1/2} \quad (20)$$

An expression for N_0 can be similarly derived from Eq. (3) as follows:

$$N_{0\alpha} = \frac{\omega_{0\alpha}}{2\pi}\left\{\frac{R_{\beta^6} + 4\zeta_\alpha{}^2[1 - (1/\gamma_\alpha)]^2 R_{\beta^4}}{R_{\beta^4} + 4\zeta_\alpha{}^2[1 - (1/\gamma_\alpha)]^2 R_{\beta^2}}\right\}^{1/2} \quad (21)$$

Similar expressions may be derived for the vertical tail side load:

$$\bar{A}_{yVT} = \{[qS(C_{y\beta})_{VT}]/U\}[R_{\beta^4} + C_2{}^2 R_{\beta^2}]^{1/2} \quad (22)$$

$$N_{0VT} = (\omega_{0\beta}/2\pi)[(R_{\beta^6} + C_2{}^2 R_{\beta^4})/(R_{\beta^4} + C_2{}^2 R_{\beta^2})]^{1/2} \quad (23)$$

It should be noted that the response integral formulation for pitch angle, horizontal tail load, lateral load factor, and yaw angle is also possible. The presentation of the numerical values of the response integrals will now be discussed.

Parametric Charts

The integrals of Eq. (19) were evaluated numerically for the following ranges of the response parameters: $0.004 \leq ak_0 \leq 0.4$, $1 \leq sk_0 \leq 1000$, and $0.005 \leq \zeta \leq 0.5$. The results are presented as parametric charts in Figs. 1–3. In the figures the response integrals are shown plotted as functions of sk_0 with ζ as a parameter and ak_0 held constant, thus forming a family of curves. Next, ak_0 is varied while ζ is held constant. As a result, the composite curves are seen to consist of families of curves together with subfamilies. The behavior of the integrals R_{β^2} and R_{β^4} is such that a three-decade reasonably dense set of k_0 values could be given without overlapping, thus permitting a visual interpolation rather than tedious cross plotting for intermediate values. However, to achieve the same clarity for R_{β^6}, that is, curve separation, the number of families had to be reduced. A word of caution must be expressed in connection with the R_{β^6} functions. The values plotted are not obtained by integrating over-all frequencies from $0 \to \infty$ because the integral R_{β^6} converges very slowly. The upper integration limit was chosen to be that value at which the R_{β^4} function ceased to increase appreciably. This practice is based on the belief that energy input to the system at high frequencies is not realistic.

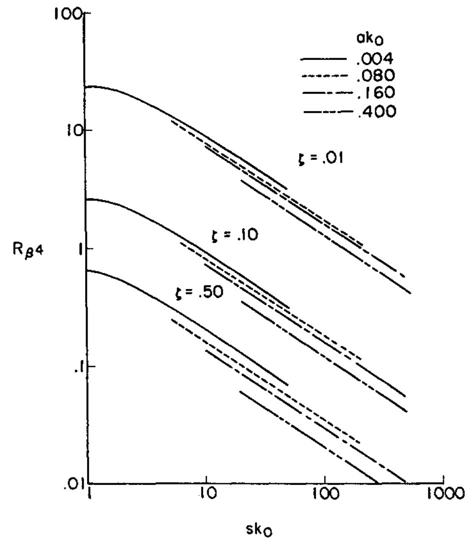

Fig. 2 Response integral, R_{β^3}.

Procedure for Response Calculation

To summarize, general formulas have been given for calculating the responses of small rigid aircraft to atmospheric turbulence. These responses consist of the ratio of rms response to rms turbulence velocity $\bar{A}$ and average number of positive slope zero crossings per unit time N_0. The formulas contain "response integrals" that are functions of stability parameters and gust scale describing the behavior of the rigid-body modes of the airplane. Values of these response integrals are obtained from a series of charts included herein. It is the purpose of this section to describe the steps for obtaining $\bar{A}$'s and N_0's for normal load factor due to vertical gust components and vertical tail load due to lateral components.

Input Parameters

The procedure is as follows: the geometric and inertial properties of the aircraft (S, $\bar{c}$, b, I_{yy}, etc.), its stability derivatives ($C_{L\alpha}$, $C_{m\alpha}$ $C_{m\dot{\alpha}}$, C_{m_q}, $C_{y\beta}$, etc.), and the flight conditions (U, q, Mach number) are required. With these data, the various force coefficients (Z_w, M_w, Y_v, N_r, etc.) defined in Eqs. (5) and (13) are computed.

The longitudinal and lateral stability characteristics are obtained from the relationship defined with Eqs. (9) and (15).

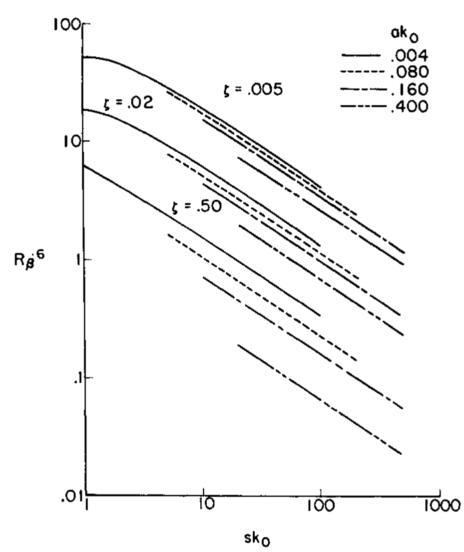

Fig. 3 Response integral, R_{β^4}.

Table 1 Aircraft flight conditions and stability derivatives

U = 244 m/sec	$C_{L\alpha}$ = 5.17	$(C_{n_r})_{VT}$ = −0.1573	$C_{n\beta}$ = 0.11699	I_{yy} = 39600 kg-m^2
ρ = 0.00131	$C_{m\alpha}$ = −1.03	$(C_{y\beta})_{VT}$ = −0.50539	C_{n_r} = −0.14049	q = 20000 N/m^2
m = 7880 kg	$C_{m\dot{\alpha}}$ = −3.2	$(C_{y_r})_{VT}$ = 0.418	C_{y_r} = 0.394	a = 0.8
I_{zz} = 69800 kg-m^2	C_{m_q} = −6.8	$(C_{n\beta})_{VT}$ = 0.1146	b = 13.6 m	L = 764 m
S = 31.8 m^2	$C_{y\beta}$ = −0.7918	W = 26000 N	$\bar{c}$ = 2.6 m	

As derived earlier, the gust parameters required are the nondimensional gust scale $s = 2L/\bar{c}$ and the unsteady lift function attenuation factor a. The gust scale s will depend on the value of the gust length L employed. An L of 764 m had been suggested for altitudes above 764 m^2. The unsteady lift function attenuation factor a depends on the wing planform and Mach number for the longitudinal case or the fuselage-fin geometry for the lateral case. Numerical values of the response integrals R_{β^2}, R_{β^4}, and R_{β^6} are obtained from the charts on Figs. 1–3, corresponding to value of the dimensionless quantities (ak_0, ζ, sk_0). Reasonably accurate values of the response integrals may be read directly from the parametric charts for particular values of sk_0 and ak_0. It will most likely happen that the actual values of the parameters will differ from those upon which the charts are based. In this case, intermediate points may be found by direct visual interpolation utilizing a flexible log scale (a strip cut from log graph paper) or by constructing auxiliary cross plots.

Sample Calculations

Application of the foregoing procedure is illustrated by the following sample calculations based on an aircraft whose characteristics are given in Table 1. The intermediate steps involved in computing the force coefficients will be omitted.

Normal load factor

From the data in Table 1, the longitudinal stability characteristics are found to be $k_{0\alpha} = 0.035$, $\zeta_\alpha = 0.29$, $\gamma_\alpha = 2.25$, and $sk_0 = 20.8$. Values for the response integrals R_{β^2} and R_{β^4} are taken from Figs. 1 and 2 by interpolating between $\zeta = 0$ and $\zeta = 0.5$, and are $R_{\beta^4} = 0.185$ and $R_{\beta^2} = 0.155$, respectively. A value of $\bar{A}_n = 0.079$ g/m/sec is then obtained from Eq. (20).

The number of positive slope zero crossings, $N_{0_n} = 2.4$ crossings/sec, is similarly calculated from Eq. (21) using a value of $R_{\beta^6} = 1.0$ from Fig. 3.

Vertical tail side load

The calculations for the vertical tail side load are carried out as previously using the stability derivatives as obtained from equations given in Ref. 8. In addition, the vertical tail surface aerodynamic characteristics and aircraft yaw radius of gyration are required for the tail load parameters. Using the data in Table 1, the following derivatives are derived: $k_{0\alpha} = 0.02$, $\zeta_\beta = 0.098$, $\gamma_\beta = 2.83$, $sk_0 = 12.0$.

Values of the response integrals corresponding to these lateral derivatives are $R_{\beta^2} = 0.78$, $R_{\beta^4} = 0.82$, and $R_{\beta^6} = 2.4$.

The tail load parameter C_2 in Eq. (22) is given by the equation

$$C_2 = -2\zeta_\beta\left(1 - \frac{1}{\gamma_\beta}\right) + \frac{k_{0\beta}b}{\bar{c}}\left(\frac{C_{y_r}}{C_{y\beta}}\right)_{VT}\left[4\,\frac{\zeta_\beta^2}{\gamma_\beta}\left(1 - \frac{1}{\gamma_\beta}\right) - 1\right]$$

and has a value of −0.0395. Equations (22) and (23) are then employed to obtain $\bar{A}_{yVT} = 369$ N/m/sec and $N_{0VT} = 1.04$ crossings/sec, respectively.

Table 2 Gust response formulas discrete vs continuous

Continuous	Discrete
Normal load factor	
$\sigma\Delta_n = \sigma_Z/W$	$\Delta n = Z_{\text{gust}}/W$
$\sigma_Z = qS_w(C_{L\alpha})_A K_\Phi \sigma_\alpha$	$Z_{\text{gust}} = qS_W(C_{L\alpha})_A K_g \alpha_{de}$
$\sigma_\alpha = \sigma_w/U$	$\alpha_{de} = U_{de}/U_e$
$K_\Phi = K_\Phi(k_{0\alpha}, \gamma_\alpha, \zeta_\alpha, sk_{0\alpha})$	$K_g = \dfrac{0.88\mu_g}{5.3 + \mu g}$
$= \{R_{\beta^4} + 4\zeta^2[1 - (1/\gamma)]^2 R_{\beta^2}\}^{1/2}$	$\mu_g = \dfrac{2Wg}{\rho CS(C_{L\alpha})_a}$
Vertical tail side load	
$\sigma_{YVT} = qS(C_{y\beta})K_{VT}\sigma_\beta$	$(Y)_{VT} = qS_{VT}(C_{y\beta})_{VT}K_{gVT}\beta_{de}$
$\sigma_\beta = \sigma_v/U$	$\beta_{de} = U_{de}/U_e$
$K_{VT} = (C_1^2 R_{\beta^4} + C_2^2 R_{\beta 2})^{1/2}$	$K_{gt} = \dfrac{0.88\mu_{gt}}{5.4_{gt} + \mu_{gt}}$
	$\mu_{gt} = \dfrac{2Wg}{\rho C_{av} S_t a_t}\left(\dfrac{K}{L_t}\right)^2$

Spectral and Discrete Gust Techniques Compared

A comparison between the spectral technique and the discrete gust approach is given in Ref. 5. It is shown that the

$$K_\Phi = \frac{2W}{\rho USC_{L\alpha}}\left\{\int_0^\infty \Phi_w(\Omega)[\Delta n(\Omega)]^2 d\Omega/\sigma_w{}^2{}_g\right\}^{1/2}$$

and that

$$\bar{A} = (\rho USC_{L\alpha}/2W)K_\Phi$$

is analogous to the gust-alleviation factor K_g of Ref. 1. The spectral gust-alleviation factors may be associated with vertical motion, pitching motion, and lateral motion. The factor K_g was originally derived for a nonpitching vertically translating airplane only.

Reference 12 specifies that the factor K_g (with modifications) be used for both vertical and lateral loads. In this paper,

$$\bar{A}_n = |2\zeta_\alpha\omega_{0\alpha}/g\gamma_\alpha|\{R_{\beta^4} + 4\zeta_\alpha{}^2[1 - (1/\gamma_\alpha)]^2 R_{\beta^2}\}^{1/2}$$

where

$$2\zeta_\alpha\omega_{0\alpha}/g\gamma_\alpha = \rho USC_{L\alpha}/2W$$

so that

$$K_\Phi = \{R_{\beta^4} + 4\zeta_\alpha{}^2[1 - (1/\gamma_\alpha)]^2 R_{\beta^2}\}^{1/2}$$

Similarity of form between discrete and spectral formulas used to derive the normal load factor and the vertical tail side load response factors is shown in Table 2. It should be kept in mind, however, that discrete gust calculations yield peak response values whereas continuous gust calculations yield rms response values.

Concluding Remarks

A procedure for estimating the longitudinal and lateral response of small rigid aircraft to independently applied vertical and lateral random turbulence has been presented. The calculation of the response in each case is based on a two-degree-of-freedom system. The procedure uses parametric charts of "response integrals." These integrals, which are functions of the undamped reduced frequency, the damping ratio, and the gust to airplane scale parameter have been evaluated for a numerical range of the parameters representative of small rigid aircraft. Because of the regular behavior of the response integral functions, reasonably accurate estimates of the aircraft response quantities can be readily and simply obtained by use of the charts.

References

[1] Pratt, K. G. and Walker, W. G., "A Revised Gust Load Formula and a Reevaluation of V-G Data Taken on Civil Transport Airplanes From 1933 to 1950," Rept. 1206, 1954, NACA.

[2] Hoblit, F. M. et al., "Development of a Power-Spectral Gust Design Procedure for Civil Aircraft," ADS-53, Jan. 1966, Federal Aviation Administration.

[3] Fuller, J. R. et al., "Contributions to the Development of a Power-Spectral Gust Design Procedure for Civil Aircraft," ADS-54, Jan. 1966, Federal Aviation Administration.

[4] Pratt, K. G. and Bennett, F. V., "Charts for Estimating the Effects of Short-Period Stability Characteristics of Airplane Vertical-Acceleration and Pitch-Angle Response in Continuous Atmospheric Turbulence," TN 3992, 1967, NASA.

[5] Houbolt, J. C., Steiner, R., and Pratt, K. G., "Dynamic Response of Airplanes to Atmospheric Turbulence Including Flight Data on Input and Output Response." TR R-199, 1964, NASA.

[6] "Fundamentals of Design of Piloted Aircraft Flight Control Systems," *Dynamics of the Airframe*, Vol. II, AE-614, Feb. 1953, Bureau of Aeronautics.

[7] Bozhar, J. and Davis, P., "Consideration of Dynamic Loads on the Vertical Tail by the Theory of Flat Yawing Maneuvers," TR-838, 1946, NACA.

[8] Eggleston, J. M. and Phillips, W. H., "The Lateral Response of Airplanes to Random Atmospheric Turbulence," TR R-74, 1960, NASA.

[9] Funk, J. and Rhyne, R. J., "An Investigation of the Loads on the Vertical Tail of a Jet-Bomber Airplane Resulting From Flight Through Rough Air," TN 3741, Oct. 1956, NACA.

[10] Diederich, F. W., "The Response of an Airplane to Random Atmospheric Disturbances," Rept. 1345, 1958, NACA.

[11] Watkins, C. E., Woolston, D. S., and Cunningham, H. J., "A Systematic Kernel Function Procedure for Determining Aerodynamic Forces on Oscillating or Steady Finite Wings at Subsonic Speeds," TR R-48, 1959, NASA.

[12] "Airworthiness Standards—Transport Category Airplanes," *Federal Aviation Regulations*, Federal Aviation Agency, Feb. 1965, Pt. 25.

SYNOPTIC: Optimization of Airfoils for Maximum Lift, R. H. Liebeck, Douglas Aircraft Company, Long Beach, Calif. and A. I. Ormsbee, University of Illinois, Urbana, Ill.; *Journal of Aircraft*, Vol. 7, No. 5, pp. 409–415.

Airplane and Component Aerodynamics Rotary Wing and VTOL Aerodynamics

Theme

A method is presented for computing the shape and lift coefficient of single element airfoils (no flaps or slots) designed specifically to achieve high lift coefficients without flow separation. The method involves two-dimensional incompressible flow theory. Example airfoil shapes and their lift coefficients, computed by this method, are presented.

Content

The basic problem solved is that of designing an airfoil which obtains maximum possible lift in an incompressible flow. The approach used is to develop a basic form for the pressure distribution which does not separate and provides maximum lift. This pressure distribution is then modified sufficiently so that it corresponds to a realistic and practical airfoil shape.

The lift coefficient in terms of the pressure distribution can be written in the form

$$C_L = \int_0^1 C_p \frac{dx}{c}\bigg|_{\substack{\text{airfoil}\\\text{lower}\\\text{surface}}} - \int_0^1 C_p \frac{dx}{c}\bigg|_{\substack{\text{airfoil}\\\text{upper}\\\text{surface}}}$$

where the freestream is aligned with the x axis. The lower-surface pressure distribution is limited by stagnation ($C_p = 1$) which evidently can actually only occur at a single point near the leading edge. The upper-surface pressure distribution is limited by boundary-layer separation, and Stratford's separation theory (Ref. 1) together with the calculus of variations is used to determine the upper-surface distribution minimizes the second integral in the previous equation while the flow remains unseparated over the entire surface.

Weber's second-order inverse airfoil theory (Ref. 2) is used to obtain the airfoil profiles corresponding to the optimized pressure distributions. Some modification of the distributions is required in order to obtain an airfoil which is reasonably thick and closes at the trailing edge.

A typical result is shown in the Fig. 1. In this case, it is assumed that the boundary layer remains laminar on the upper surface until it reaches the adverse pressure gradient, at which time instantaneous transition occurs. A set of airfoils for freestream Reynolds numbers of one, five, and ten million for both the cases of a laminar rooftop and a fully turbulent rooftop have been designed and are presented in the paper. For the airfoil shown in the figure, additional calculations have been made using the Douglas Neumann Potential Flow Program, and the Douglas Turbulent Boundary Layer Program. They verified that the profile does provide the desired pressure distribution, and that the boundary layer does not separate.

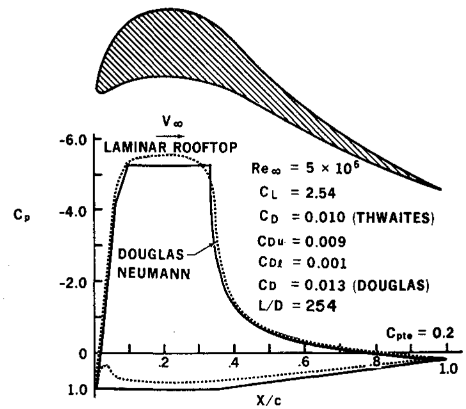

Fig. 1 Airfoil geometry and pressure distribution, laminar rooftop, $Re_\infty = 5 \times 10^6$.

References

[1] Stratford, B. S., "The Prediction of Separation of the Turbulent Boundary Layer," *Journal of Fluid Mechanics*, Vol. 5, 1959.

[2] Weber, J., "The Calculation of the Pressure Distribution on the Surface of Thick Cambered Wings and the Design of Wings with Given Pressure Distribution," R and M No. 3026, June 1955, Aeronautical Research Council, London, England.